

**Optimization of Weight On Bit During Drilling Operation Based on Rate of
Penetration Model**

by

Adib Mahfuz B And Rahman

Dissertation submitted in partial fulfillment of
the requirements for the
Bachelor of Engineering (Hons)
(Petroleum Engineering)

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**Universiti Teknologi PETRONAS
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CERTIFICATION OF APPROVAL

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A project dissertation submitted to the
Petroleum Engineering Programme
Universiti Teknologi PETRONAS
in partial fulfillment of the requirement for the
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January 2011

CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the references and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.



ADIB MAHFUZ BIN ABD RAHMAN

ABSTRACT

Drilling optimization is very important during drilling operation. Optimization of drilling could save time and cost of operation thus increases the profit. Drilling optimization aims to optimize controllable variables during drilling operation such as weight on bit and bit rotation speed for obtaining optimum drilling rate. In this project, Bourgoyne and Young ROP model has been selected to study the effects of several parameters during drilling operation. Important parameters such as depth, pore pressure, equivalent circulating density, bit weight, rotary speed, bit tooth wear, and jet impact force are extracted from drilling report. In order to study their relationship statistical method which is multiple regressions analysis has been used. The penetration model for the field is constructed using the results of statistical method. In the end, the result from analysis is used to determine optimum values of weight on bit that give optimize drilling operation.

Overall, this project provides a study to the most complete mathematical model for rate of penetration that was constructed by Bourgoyne and Young. From the research the constants that represented several drilling variables had been determined. The rate of penetration for the field had been predicted based on the constants for every data depth. Finally, the optimized weight on bit had been calculated for the several data points and the results had been simulated using drilling simulator.

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NOMENCLATURE

Roman

<i>Symbol</i>		<i>Unit, Field (SI)</i>
a_1	formation strength parameter	-
a_2	normal compaction exponent	-
a_3	under compaction exponent	-
a_4	pressure differential exponent	-
a_5	bit weight exponent	-
a_6	rotary speed exponent	-
a_7	tooth wear exponent	-
a_8	hydraulic exponent	-
D	depth of borehole	[L], ft (m)
d_b	diameter of bit	[L], in (mm)
d_n	equivalent bit nozzle diameter	[L], in (mm)
f_1	formation strength function	-
f_2	formation compaction function	-
f_4	pressure differential of hole bottom function	-
f_5	bit diameter and weight function	-
f_6	rotary speed function	-
f_7	tooth wear function	-
f_8	hydraulic function	-
g_p	pore pressure gradient of the formation	[M/L ³], ppg (sg)
h	bit tooth dullness, fractional tooth weight worn away	-
H_1	constants for tooth geometry of bit types	-

N	<i>rotary speed</i>	$[T^{-1}]$, rpm (-)
R	<i>rate of penetration</i>	-
x_1	<i>drilling rate of penetration independent parameter</i>	-
x_2	<i>normal compaction drilling parameter</i>	-
x_3	<i>under-compaction drilling parameter</i>	-
x_4	<i>pressure differential drilling parameter</i>	-
x_5	<i>bit weight drilling parameter</i>	-
x_6	<i>rotary speed drilling parameter</i>	-
x_7	<i>tooth wear drilling parameter</i>	-
x_8	<i>bit hydraulics drilling parameter</i>	-
W	<i>weight on bit</i>	$[ML/T^2]$, 1000 lbf
W/d	<i>weight on bit per inch of bit diameter</i>	$[M/T^2]$, 1000 lbf/in (N/m)
$(W/d)_{max}$	<i>max weight on bit</i>	$[M/T^2]$, 1000lbf/in (N/m)
$(W/d)_t$	<i>threshold bit weight</i>	$[M/T^2]$, 1000 lbf/in (N/m)

Greek

Symbol		Unit, Field (SI)
P	drilling fluid density	$[M/T^3], \text{ppg}(\text{kg}/\text{m}^3)$
P_c	equivalent circulating mud density	$[M/L^3], \text{ppg}(\text{sg})$
μ	apparent viscosity at $10,000 \text{ sec}^{-1}$	$[M/LT], \text{cp}(\text{Pa s})$

Abbreviations

Symbol		Unit, Field (SI)
BHA	bottom hole assembly	-
ECD	equivalent circulating density	$[M/L^3], \text{ppg}(\text{sg})$
MD	measured depth	$[L], \text{ft}(\text{m})$
MW	mud weight (density)	$[M/L^3], \text{PPg}(\text{sg})$
Opt	optimum	-
PDC	polycrystalline diamond cutter	-
PPFG	pore pressure fracture gradient	-
ROP	rate of penetration	$[L/T], \text{ft/hr}(\text{m/hr})$
RPM	revolution per minute	$[T^{-1}], \text{rpm}(-)$
TD	total depth	$[L], \text{ft}(\text{m})$
TVD	true vertical depth	$[L], \text{ft}(\text{m})$
WOB	weight on bit	$[ML/T^2], 1000 \text{ LBF}$

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF STUDY

Development of oilfield is subject to drill in cost efficient manners. For that reason oilfield drilling operations will face hurdles to reduce overall costs, increase performances and reduce the probability of encountering problems. The increase in complexity for drilling operation has increase many problems thus result in critical cost consideration. Different methods from different disciplines are being used nowadays in drilling activities in order to obtain a safe, environmental friendly and cost effective well construction.

Optimization of drilling operation can be obtained by increasing drilling speed. In drilling industry, the first well drilled in a new field generally will have the highest cost. With increasing familiarity to the area optimized drilling could be implemented those decreasing costs of each subsequent well to be drilled until a point is reached at which there is no significance improvement. The relationship among drilling parameters are complex, however determination of optimize operating conditions will result in minimum cost of drilling.

Major drilling variables considered to have an effect on drilling rate of penetration are not fully comprehend and complex to model. There are many proposed mathematical models which attempted to combine known relations of drilling parameters. The proposed models worked to optimize drilling operation by mean of selecting the best bit weight and rotary speed to achieve minimum cost. Considerable drilling cost reductions have been achieved by means of using the available mathematical models.

1.2 PROBLEM STATEMENT

It is very important to understand the relationship between drilling parameters. Though it is very hard to provide the relationship of all parameters but with proper application of mathematical method estimation could be predicted those at least can be the guide for planning optimization of drilling operation.

Drilling parameters that is considered throughout this study include formation strength and bit type, formation depth, pore pressure, equivalent circulating density, bit weight, rotary speed, bit tooth wear and jet impact force. Other effects of drilling variables such as mud type, solid content are also included in term of formation strength and bit type.

This project is focusing on optimization of controllable variable which is weight on bit with consideration on other drilling parameters. Though it is important to bear in mind that formation properties, which are uncontrollable are one of the most critical factors in drilling performance determination. Drilling fluid properties and bit types, though controllable are not in good drilling practice to change in ordinary bit runs.

The benefit of statistical method is the ability of being able to estimate the rate of penetration as a function of independent drilling parameters. Following the analysis of the drilling parameters, a relation between drilling parameters and rate of penetration could be determined.

For that reasons, the ability to relate drilling parameters and the possibility to analyze it with mathematical methods provide the best ways to optimize drilling operation.

1.3 OBJECTIVE

This study aims to:

- Determine the constants a_1 until a_8 that represent several drilling parameters for the field which are drillability, normal compaction, under compaction, pressure differential, weight on bit, rotary speed, bit tooth wear and hydraulic.
- Predict rate of penetration vs. depth for the field base on the constants that have been determine.
- Determine optimize value of weight on bit specifically for certain depth in order to have optimum drilling operation.

1.4 SCOPE OF STUDIES

The project will concentrate on analyze and extract data from the drilling report to obtain variables x_1 until x_8 for each data point. Statistical analysis which is multiple regressions will be applied to the variables to determine the constants of a_1 until a_8 that represent the formation. The constant coefficient will be use to predict rate of penetration for the field. Then, the model for the field will be constructed and rate of penetration will be predicted. Next, optimum weight on bit will be calculated using the available equation. A simulation with Drill Sim 500 will be conducted to compare the results base on actual field data.

CHAPTER 2

LITERATURE REVIEW/THEORY

2.1 OVERVIEW

According to field data, there are several methods to reduce drilling costs. One of these methods is the optimization of drilling parameters to obtain the maximum rate of penetration in each bit run. Many parameters affect rate of penetration like hole cleaning (including drillstring rotation, weight on bit, floundering phenomena, mud rheology), tooth wear, formation hardness (including depth and kind of formation), differential pressure including mud weight. To optimize drilling parameters, it is required that an appropriate rate of penetration model to be selected until acceptable results are obtained. In literature, there are various applicable models to predict rate of penetration such as Bourgoyne and Young model, Bingham model and modified Warren model.

Optimization of drilling activities for oil and gas wells is an area for which numerous detailed research studies have been performed. Optimized drilling is a system of pre-selecting the magnitude of controllable drilling variables to maximize footage or minimize drilling cost [1]. Optimization of drilling operation is very critical due to increasing demand to drill wells that is more complex and problematic. Thus it is very important to have more research in this field to make sure that drilling operations could not be major impact in field development.

There are mainly three optimization methods that are well known such as Galle and Woods [3], drill-off test and statistical method (multiple regressions) [4].

2.2 DRILLING OPTIMIZATION STUDIES

One of the most important early studies performed in regards to the optimal drilling detection was by Bourgoyne and Young [4]. They constructed a linear penetration rate model and performed a multiple regression analysis of drilling data in order to select bit weight, rotary speed, and bit hydraulics. In their analysis they included the effects of formation strength, formation depth, formation compaction, pressure differential across the hole bottom, bit diameter, bit weight, rotary speed, bit wear and bit hydraulics. They found that regression analysis procedure can be used to systematically evaluate many of the constants in the penetration rate equation.

Maurer [5] derived rate of penetration equation for roller-cone type of bits considering the rock cratering mechanisms. The equation was based on ‘perfect cleaning’ condition where all of the rock debris is considered to be removed between tooth impacts. A working relation between drilling rate, weight on bit and string speed was achieved assuming that the hole was subject perfect hole cleaning circumstances. It was also mentioned that the obtained relationships were a function of drilling depth. The rate of drilling equation expresses as given in equation 2.1

$$\frac{dF}{dt} = \frac{4}{\pi d_b^2} \frac{dV}{dt} \dots\dots\dots (2.1)$$

where, f is the distance drilled by bit, t is time, V is volume of rock removed, and d_b is diameter of the bit.

Bingham [6] proposed a rate of penetration equation based on laboratory data, equation 2.2. In their equation the threshold bit weight was assumed to be negligible and rate of penetration was a function of applied weight on bit and rotary speed of string. The bit weight exponent, a₅ was set to be determined experimentally through the prevailing conditions.

$$R = K \left(\frac{WOB}{d_b} \right)^{a_5} N \dots\dots\dots (2.2)$$

Young [7] performed development of onsite computer system to control bit weight and rotary speed. He introduced a minimum cost drilling terminology with four main

equations; drilling rate as a function of weight on bit and bit tooth height, bit wearing rate as a function of bit rotation speed, bit tooth wear rate and finally drilling cost. By integration of the introduced equations for the optimum weight on bit and rotary speed constants the best solutions are reported to be obtained.

Al-Betairi et al. [8] presented a case study for optimizing drilling operations in the Arabian Gulf area. The drilling model proposed by Bourgoyne and Young [9] was applied in their model with Statistical Analysis System was validated. They observed that for particular set of coefficients of the model was observed to be inversely proportional to the influence of that parameter on the rate of penetration. The more the data points the reliable estimated drilling parameters became.

Warren [10] presented an ROP model that includes the effect of both the initial chip generation and cuttings-removal process. The rate of penetration equation they derived is formed of two terms, working only with perfect hole cleaning assumption. The first term defined the maximum rate supporting the WOB effect without tooth penetration rate, the second term on the other hand considering tooth penetration into the formation. The equation was found to fit the experimental data for both steel tooth and insert bit types.

Miska [11] presented three governing differential equations; rate of penetration, rate of tooth wear, and rate of bearings wear. It was concluded that given equations could have been successfully used for predicting and optimization purposes provided that some major conditions were satisfied. Three major conditions could be listed as; bottom hole cleaning is adequate, rock-bit is properly selected to the formation drilled, and formation can be considered macroscopic-homogeneous.

Akgun[12] investigated the controllable drilling variables having effect on drilling rate. Mud weight, weight on bit, rotary speed, bit type and hydraulics are among the controllable drilling variables. Proper selection of the controllable variables is reported to significantly improve drilling rate. An upper drilling rate limit or “technical limit” concept has been introduced which can not be exceeded without risking the drilling operations safety. For example selected mud weight not less than the weight which

could result in well kick and borehole collapse (wellbore stability). WOB and RPM parameters need to be at maximum possible values considering the minimum bit operational cost and drillstring stability. Flow rate need to be selected at an optimum value by considering bit hydraulics and hole cleaning.

Ozbayoglu and Omurlu [13] performed a study in which they mathematically optimized drilling parameters in order to reduce the drilling costs. They considered that weight on bit, rotation speed, bit type and wear, and bit hydraulics have direct impact on rate of penetration. An analytical drilling cost equation was defined considering a non-linear rate of penetration equation. Drilling parameters of the actual field data collected from the literature were optimized using the defined equation by means of certain mathematical models. They observed that drilling costs were reduced up to four times.

2.3 FACTOR AFFECTING RATE OF PENETRATION

Rate of penetration is affected by several variables. This includes control and uncontrolled variables. The most important variables affecting penetration rate that have been identified and studied include bit type, formation characteristics, drilling fluid properties, bit operating conditions (bit weight and rotary speed), bit tooth wear, and bit hydraulics [2].

These factors are proven based on some experimental work which study the effect of these variables on drilling rate that has been done by several authors. In most of this experimental work, the effect of as single variable was studied while holding other variables constant.

2.3.1 Bit Type

The bit type selected has a large effect on penetration rate. For rolling cutter bits, the initial penetration rate is often highest in a given formation when using bits with long teeth and large cone offset angle. However, these bits are practical only in soft formations because of rapid tooth destruction and decline on penetration rate in hard formations. The lowest cost per foot drilled usually is obtained when using the longest

tooth bit that will give a tooth life consistent with the bearing life at optimum bit operating condition.

Drag bits are designed to obtain a given penetration rate. Drag bits give a wedging-type rock failure in which the bit penetration per revolution depends on the number of blades and the bottom cutting angle. The diamond and PCD bits are designed for a given penetration per revolution depends on the number of blades and the bottom cutting angle. The diamond and PCD bits are designed for a given penetration per revolution by the selection of the size and number of diamonds or PCD blanks. The width and number of cutters can be used to compute the effective number of blades. The length of the cutters projecting from the face of the bit (less the bottom clearance) limits the depth of the cut.

2.3.2 Formation Characteristics

The elastic limit and ultimate strength of the formation are the most important formation properties affecting penetration rate. The shear strength predicted by the Mohr failure criteria sometimes is used to characterize the strength of the formation. Maurer [5] has reported that the crater volume produced beneath a single tooth is inversely proportional to both the compressive strength of the rock and the shear strength of the rock. Bingham [6] found that the threshold force required to initiate drilling in a given rock at atmospheric pressure could be correlated to the shear strength of the rock as determined in a compression test at atmospheric pressure.

The permeability of the formation also has a significant effect on penetration rate. In permeable rocks, the drilling fluid filtrate can move into the rock ahead of the bit and equalize the pressure differential acting on the chips formed beneath each tooth. This would tend to promote the more explosive elastic mode of crater formation. It is also can be argued that the nature of the fluids contained in the pore spaces of the rock also affects this mechanisms since more filtrate volume would be required to equalize the pressure in a rock containing gas than in a rock containing liquid.

The mineral composition of the rock also has some effect on penetration rate. Rocks containing hard, abrasive minerals can cause rapid dulling of the bit teeth. Rocks

containing gummy clay minerals can cause the bit to ball up and drill in a very inefficient manner.

2.3.3 Drilling fluid Properties

The properties of the drilling fluid reported to affect the penetration rate include density, rheological flow properties, filtration characteristics, solid content and size distribution, and chemical composition.

Penetration rate tends to decrease with increasing fluid density, viscosity, and solids content, and tends to increase with increasing filtration rate. The density, solids content, and filtration characteristics of the mud control the pressure differential across the zone of crushed rock beneath the bit. The fluid viscosity controls the parasitic frictional losses in the drillstring and, thus, the hydraulic energy available at the bit jets for cleaning. There is also experimental evidence [14] that increasing viscosity reduces penetration rate even when the bit is perfectly clean. The chemical composition of the fluid has an effect on penetration rate in that the hydration rate and bit balling tendency of some clays are affected by the chemical composition of the fluid.

The effect of drilling fluid density and the resulting bottomhole pressure on penetration rate has been studied by several authors. Experiment by Maurer [5], which were conducted using a single bit tooth under simulated borehole conditions have provided some insight into the mechanism by which an increase in drilling fluid density causes a decrease in penetration rate for rolling cutter bits. An increase in drilling fluid density causes an increase in the bottomhole pressure beneath the bit and, thus, an increase in the pressure differential between the borehole pressure and the formation fluid pressure. This pressure differential between the borehole pressure and formation fluid pressure often is called the overbalance.

Cunningham and Eenink, [15] working with a 1.25-in.-diameter rolling cutter bit in a laboratory drilling machine, studied the effect of overbalance on penetration rate for a wide range of rock permeabilities. Note that a good correlation is obtained when the data are replotted with drilling rate as function of overbalance ($p_{bh} - p_f$). Apparently, formation damage beneath the bit caused by the deposition of a filter cake of mud and

formation solids prevented a flow of mud filtrate ahead of the bit sufficient to equalize the pressure differential. The effect of overbalance on penetration rate is more pronounced at a low value of overbalance than at high value of overbalance. If the overbalance is quite large, additional increases in overbalance have essentially no effect on penetration rate.

Garnier and van Lingen [16] have published laboratory data obtained using both small drag bits and rolling cutter bits in a laboratory drilling apparatus. They concluded that the effective overbalance during chip removal by a drag bit often can be greater than the difference between the static borehole and rock pore pressure. When a chip is being lifted, a vacuum can be created under the chip unless sufficient liquid can be supplied to fill the opening void space. The liquid can be supplied only by (1) drilling fluid flowing through the fracture, (2) drilling fluid filtrate flowing through the pores of the chip, and (3) formation fluid flowing into the void from the rock beneath the chip. When drilling a rock of low permeability with a clay/water mud which readily forms a filter cake, the flow of liquid into the void beneath the chip was found to be too slow to prevent a pressure reduction beneath the chip. When the mud is used as the drilling fluid, penetration rate decreased with increasing mud pressure, even though the static overbalance remained constant. This indicates that the effective dynamic overbalance during the chip formation was greater than the static overbalance. When water was used as the drilling fluid, pressure equalization beneath the chip was more rapid for the rocks of moderate permeability, and penetration rate remain constant with increasing mud pressure.

To obtain the effect of overbalance on penetration rate for a drag bit, Garnier and Van Lingen operated their machine at various levels of the borehole pressure while maintaining the pore pressure constant at atmospheric pressure. Since the pore pressure was already quite low, the dynamic and static overbalances were essentially equal.

Some field data on the effect of overbalance on penetration rate are also available. The effect of overbalance on penetration rate in shale on seven wells drilled in south Louisiana was studied by Vidrine and Benit. [17] The shape of curve is quite similar to the laboratory data of Cunningham and Eenink. This type of behavior is

accepted widely by field drilling personnel familiar with changes in penetration rate due to changes in mud density.

Bourgoyne and Young [4] observed that the relation between overpressure and penetration rate could be represented approximately by a straight line on semi log paper for the range of overbalance commonly used in field practice. In addition, they suggested normalizing the penetration rate data by dividing by the penetration rate corresponding to zero overbalance (borehole pressure equal to formation fluid pressure). A reasonably accurate straight-line representation of the data is possible for moderate values of overbalance.

2.3.4 Operating Conditions

The effect of bit weight and rotary speed on penetration rate has been studied by numerous authors both in the laboratory and in the field. Typically, a plot of penetration rate vs. bit weight obtained experimentally with all other drilling variables held constant has the characteristic shape shown in Figure 1. No significant penetration rate is obtained until the threshold bit weight is applied (Point a). Penetration rate then increases rapidly with increasing values of bit weight for moderate values of bit weight (Segment ab). A linear curve is often observed at moderate bit weight (Segment bc). However, at higher values of bit weight, subsequent increase in bit weight causes only slight improvements in penetration rate (Segment cd). In some cases, a decrease in penetration rate is observed at extremely high values of bit weight (Segment de). This type of behavior often called bit floundering. The poor response of penetration rate at high values of bit weight usually is attributed to less efficient bottomhole cleaning at higher rates of cuttings generation or to a complete penetration of the cutting element into the hole bottom.

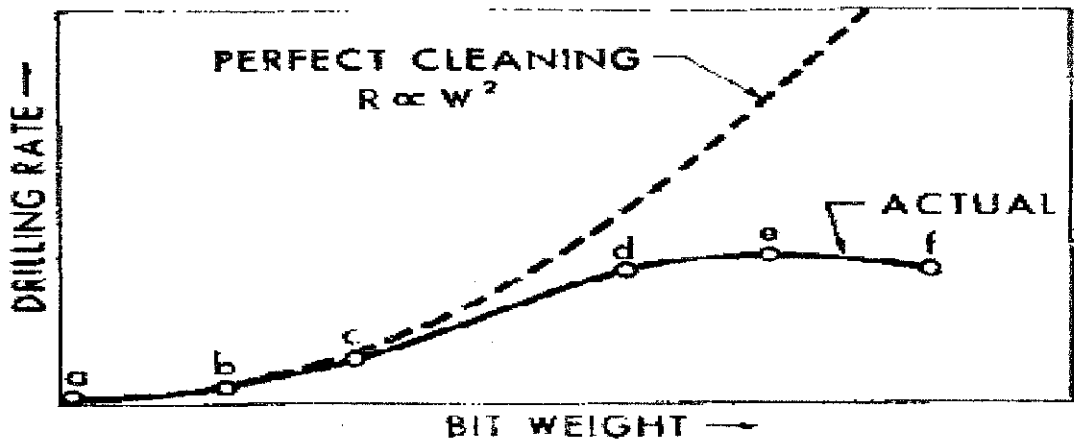


Figure 1-Typical response of penetration rate to increasing bit weight [18]

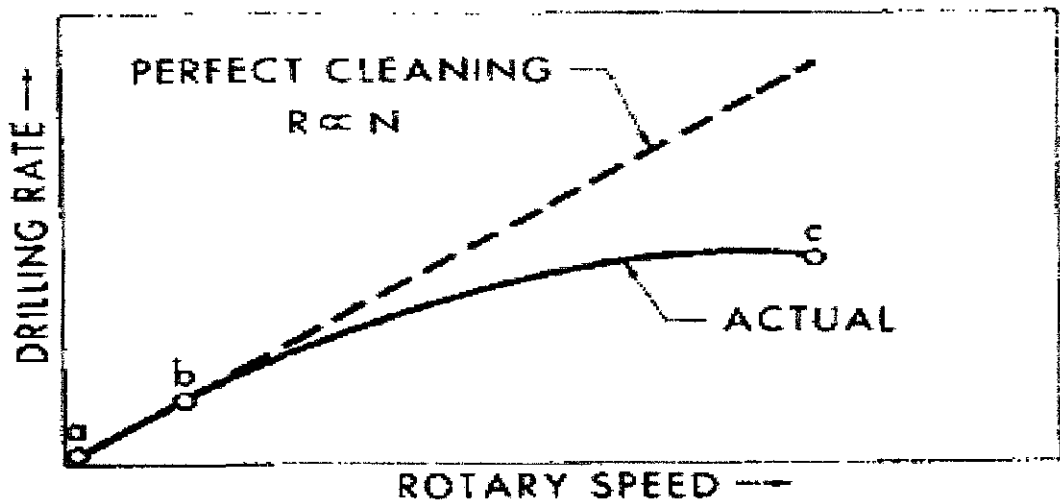


Figure 2-Typical response of penetration rate to increasing rotary speed [18]

A typical plot of penetration rate vs. rotary speed obtained with all other drilling variables held constant is shown in Figure 2. Penetration rate usually increases linearly with rotary speed at low values of rotary speed. At higher values of rotary speed, the response of penetration rate to increasing rotary speed are diminishes. The poor responses of penetration rate at high values of rotary speed usually are also attributed to less efficient bottomhole cleaning.

Maurer [5] developed a theoretical equation for rolling cutter bits relating penetration rate to bit weight, rotary speed, bit size, and rock strength. The equation was derived from the following observation made in single tooth impact experiments.

1. The crater volume is proportional to be the square of the depth of cutter penetration.
2. The depth of cutter penetration is inversely proportional to the rock strength.

For these conditions, the penetration rate R is given by

$$R = \frac{K}{s^2} \left[\frac{W}{d_b} - \left(\frac{W}{d_b} \right) t \right]^2 N \dots\dots\dots(2.3)$$

This theoretical relation assumes perfect bottomhole cleaning and incomplete bit tooth penetration.

The theoretical equation of data obtained at relatively low bit weight and rotary speeds corresponding to Segment ab in Figure 1 and Figure 2. At moderate values of bit weight, the weight exponent usually is observed to be closer to a value of one than the value of two predicted by Equation 2.3. At higher values of bit weight, a weight exponent of less than one usually is indicated. Bingham [6] suggested the following drilling equation on the basis of considerable laboratory and field data.

$$R = K \left(\frac{W}{d_b} \right)^{a_5} N \dots\dots\dots(2.4)$$

In this equation the threshold bit weight was assumed to be negligible and the bit weight exponent must be determined experimentally for the prevailing conditions. However, a constant rotary speed exponent of one was used in the Bingham equation even though some of his data showed behavior similar to that described by Segment bc in Figure 2.

More recently, several authors have proposed the determination of both a bit weight exponent and a rotary speed exponent using data representative of the prevailing conditions. Young [7] has pioneered the development of a computerized drilling control system in which both the bit weight and rotary speed could be varied systematically when a new formation type was encountered and the bit weight and rotary speed exponent automatically computed from the observed penetration rates response. Values of the bit weight exponent obtained from a filed data range from 0.6 to 2.0, while values of the rotary speed exponent range from 0.4 to 0.9.

2.3.5 Bit Tooth Wear

Most bits tend to drill slower as the bit run progresses because of tooth wear. The tooth length of milled tooth rolling cutter bits is reduced continually by abrasion and chipping. The teeth are altered by hard facing or by case-hardening process to promote a self-sharpening type of tooth wear. However, while this tends to keep the tooth pointed, it does not compensate for the reduce tooth length. The teeth of tungsten carbide insert-type rolling cutter bits fail by breaking rather than by abrasion. Often, the entire tooth is lost when breakage occurs. Reductions in penetration rate due to bit wear usually are not as severe for insert bits s for milled tooth bit unless a large number of teeth are broken during the bit run. Diamond bits also fail from breakage or loss of diamonds from the matrix.

Several authors have published mathematical models for computing the effect of tooth wear on penetration rate for rolling cutter bits. Galle and Woods [3] published the following model in 1963.

$R \propto \left(\frac{1}{0.928125h^2 + 6h + 1} \right)^{a_7}$ (2.5)

A value of 0.5 was recommended for the exponent a_7 for self-sharpening wear of milled tooth bits, the primary bit type discussed in the publication. In a more recent work, Bourgoyneand Young [4] suggested a similar but less complex relationship given by

$R \propto e^{-a_7 h}$ (2.6)

Bourgoyne and Young suggested that the exponent a_7 be determined based in the observed decline of penetration rate with tooth wear for previous bits run under similar conditions.

2.3.6 Bit Hydraulics

The introduction of the jet-type rolling cutter bits in 1953showed that significant improvements in penetration rate could be achieved through an improved jetting action at the bit. The improved jetting action promoted better cleaning of the bit teeth as well

as the hole bottom. Some evidence has been presented [18] that the jetting action is most effective when using extended-nozzle bits in which the discharge ends of the jets are brought closer to the bottom of the hole. A center jet must also be used with extended-nozzle bits to prevent bit balling in soft formations.

There is considerably uncertainty as to the best hydraulics parameter to use in characterizing the effect of hydraulics on penetration rate. Bit hydraulic horsepower, jet impact force, and nozzle velocity all are used commonly.

The level of hydraulics achieved at the bit is thought by many to affect the flounder of the bit. At low bit weights and penetration rates, the level of hydraulics required for hole cleaning is small. As more weight is applied to the bit and cuttings are generated faster, a flounder point is reached eventually where the cuttings are removed as quickly as they are generated. If the level of hydraulics is increased, a higher bit weight and penetration rate will be reached before bit floundering occurs.

Excel [18] working with microbits in a laboratory drilling machine, has made the most extensive laboratory study to date of the relation between penetration rate and the level of hydraulics. Working at constant bit weight and rotary speed, Eckel found that penetration rate could be correlated to a Reynolds number group given by

$$N_{Re} = K \frac{\rho v d}{\mu_a} \dots\dots\dots(2.7)$$

The shear rate of 10,000 *seconds*⁻¹ was chosen as representative of shear rates present in the bit nozzle. The scaling constant, K, is somewhat arbitrary, but a constant value of 1/1976 was used by Eckel to yield a convenient range of the Reynolds number group.

The results of Eckel’s experiments are summarized in Figure 3 and Figure 4. Note that penetration rate was increased by increasing the Reynolds number function for the full range of Reynolds number studied. When the bit weight was increased, the correlation curve simply was shifted upward as shown in Figure 4. The behavior at the flounder point was not studied by Eckel. It can be shown that, for a given drilling fluid, the Reynolds number function is a maximum when the jet impact force is a maximum.

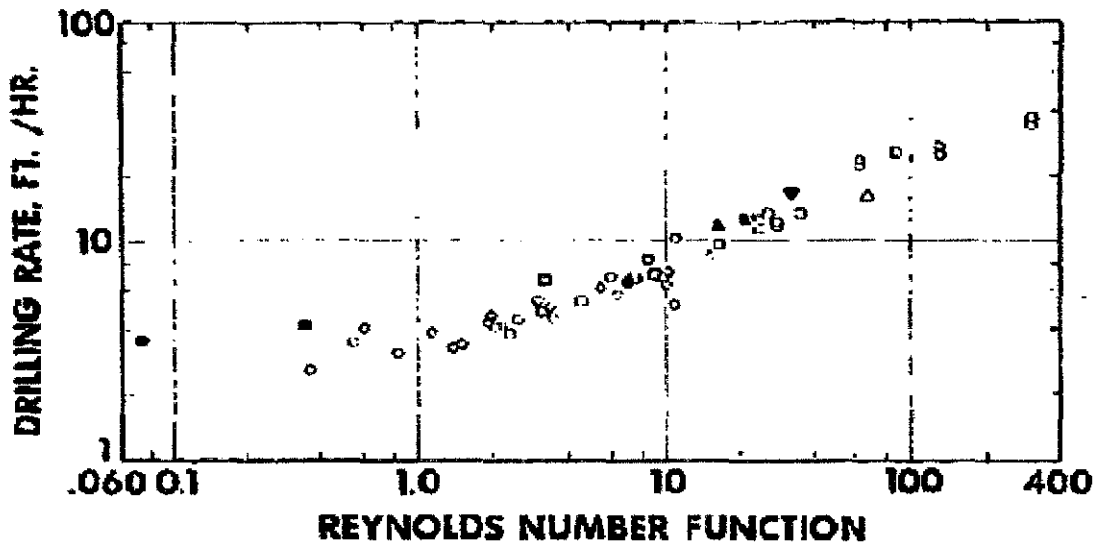


Figure 3: Penetration rates as a function of bit Reynolds number. [16]

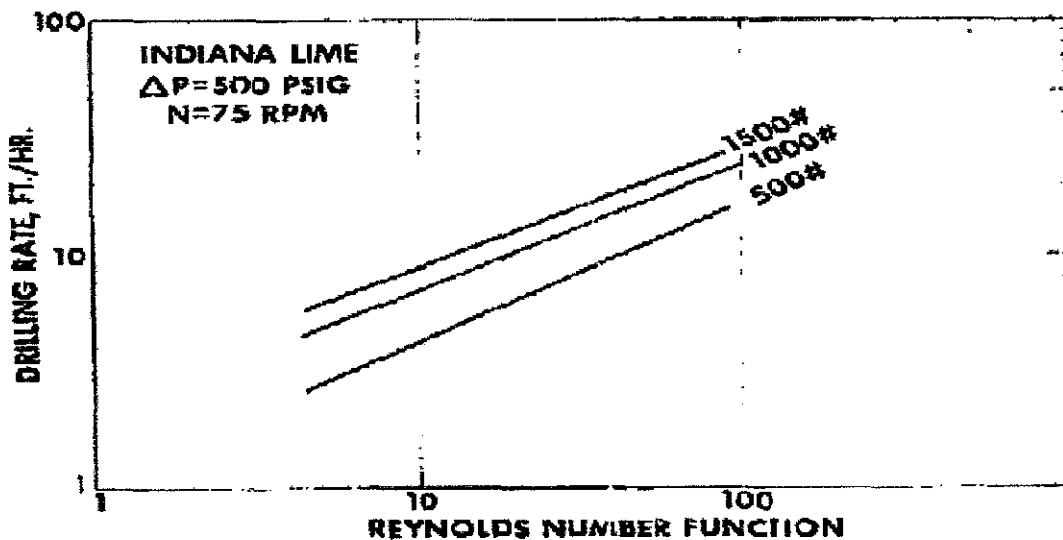


Figure 4: Experimentally observed effect of bit weight and bit Reynolds number on penetration rate. [16]

In spite of the convincing correlation presented in Figure 3 and Figure 4, Eckel's work has not been widely applied in practice. Hydraulic horsepower and jet impact force are more frequently used in the development of correlations between bit hydraulics and penetration rate. Recent data obtained in full-scale laboratory drilling experiments conducted under simulated borehole conditions [19] has shown that the jet Reynolds number group, hydraulic horsepower, and jet impact force all give similar results when use to correlate the effect of jet bit hydraulics on penetration rate.

2.4 BOURGOYNE AND YOUNGS' RATE OF PENETRATION MODEL

Bourgoyne and Youngs' [4] method is the most important drilling optimization method since it is based on statistical synthesis of the past drilling parameters. A linear penetration model is being introduced and multiple regressions analysis over rate of penetration equation is being conducted. For that reason this method is considered to be the most suitable method for drilling optimization.

The model proposed by Bourgoyne and Young [4] has been adopted for this project in order to derive equations to perform the ROP estimation using the available input data. This model has been selected because it is considered as one of the complete mathematical drilling models in use of the industry for roller-cone type of bits. Equation 2.8 gives the linear rate of penetration equation which is a function of both controllable and uncontrollable drilling variables. When the multiple regressions process is performed the model has been modified based on controllable parameters.

$$\frac{dF}{dt} = e^{(a_1 + \sum_{j=2}^8 a_j x_j)} \dots \dots \dots (2.8)$$

The normalization constants given in the general ROP equation are modified accordingly as a function of the data property when used as an input to the regression cycle. The coefficients should give accurate predictions for ROP; when modified normalization constants are used. The constants given in equation 2.8 from a_1 through a_8 should be determined through the multiple regression analysis using the drilling data. They represent the effects of formation strength, compaction effect, pressure differential, bit weight, rotary speed, tooth wear and hydraulic exponent.

The threshold bit weight on bit and bit diameter value is not constant, it significantly may have varying magnitudes based on formation characteristics, and for this reason whole data trend is observed when this threshold value is determiner as an input. The same value could easily be obtained from a drill-off test. The fractional tooth height calculation methodology is functions of reference abrasiveness constants in the same field, and is related to the time bit in use have operated. The open form of the general ROP equation for roller cone bit types is given in equation 2.9.

$$\frac{dF}{dt} = Exp(a_1 + a_2(8000 - D) + a_3(D^{0.69}(g_p - 9) + a_4(g_p - p_c) + a_5 \ln\left(\frac{w}{d_b} \frac{0.02}{4 - 0.02}\right) + a_6 \ln\left(\frac{N}{100}\right) + a_7(-h) + a_8\left(\frac{F_1}{1000}\right)) \dots (2.9)$$

The considered effects of the controllable and uncontrollable drilling variables on rate of penetration are individually described below for each item.

2.4.1 Formation Strength Function

The coefficient for the effect of formation strength is represented by “a₁”. It has been considered that the less the value for this constant, the less the penetration rate. The coefficient includes also the effects of parameters not mathematically modeled such as; the effect of drilled cuttings. Other factors which could be included for future consideration but known to be under this function could be drilling fluid details, solids content, efficiency of the rig equipment/material, crew experience, and service contractors’ efficiency.

The equation for the formation strength related effects are defined as in equation 2.1. The “f₁” term is defined in the same unit as rate of penetration, for that reason it is called drillability of the formation of interest.

$$f_1 = e^{a_1} \dots (2.10)$$

2.4.2 Formation Compaction Function

There are two functions allocated for the consideration of the formation compaction over rate of penetration. The primary function for the effect of normal compaction trend is defined by “a₂”. The primary effect of formation compaction considers an exponential decrease in penetration rate with increasing depth, as given in equation 2.11. In other means this function assumes increasing rock strength with depth due normal compaction.

$$f_2 = e^{a_2 X_2} = e^{a_2(10000 - D)} \dots (2.11)$$

The additional function considered to have an effect over the penetration rate in regards of the formation compaction is defined by the coefficient “a₃”. This function considers the effect of under compaction in abnormally pressured formation. In other means within over-pressured formations rate of penetration is going to show an increased behavior. There is an exponential increase in penetration rate with increasing pore pressure gradient, equation 2.12.

$$f_3 = e^{a_3 X_3} = e^{a_3 D^{0.69} (gp - 9.0)} \dots \dots \dots (2.12)$$

2.4.3 Pressure Differential of Bottom Hole Function

The function for the pressure differential is defined by coefficient “a₄”. Pressure differential of hole bottom function is considered to reduce penetration rate with decreasing pressure difference. Whenever the pressure differential between the hole bottom and formation is zero the effect of this function is going to be equal to 1 for the overall process, equation 2.13.

$$f_4 = e^{a_4 X_4} = e^{2.303 a_4 D (gp - pc)} \dots \dots \dots (2.13)$$

2.4.4 Bit diameter and weight function

The function for the bit diameter and weight is defined by coefficient “a₅”. The bit weight and bit diameter are considered to have direct effect over penetration rate, equation 2.14. $\left(\frac{W}{d_b}\right)t$ is the threshold bit weight, the reported values for this term ranging from 0.6 to 2.0. In this the magnitude for this term has been determined specifically based on the characteristics of the formation. The force at which fracturing begins beneath the tooth is called the threshold force. The given function is normalized for 4000 lbf per bit diameter.

$$f_5 = e^{a_5 X_5} = \left(\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)t}{4 - \left(\frac{W}{d_b}\right)t} \right)^{a_5} \dots \dots \dots (2.14)$$

2.4.5 Rotary Speed Function

The function for the rotary speed is defined by coefficient “a₆”. Likewise the direct relation of bit weight on penetration rate the rotary speed is also set to have a similar relation, equation 2.15. The normalizing value to equalize the rotary speed function to 1 is taken to be an appropriate magnitude based on the actual rotation of the bit.

$$f_6 = e^{a_6 X_6} = \left(\frac{N}{100} \right)^{a_6} \dots\dots\dots(2.15)$$

2.4.6 Tooth Wear Function

The function for the tooth wear is defined by coefficient “a₇”. The tooth wear function is calculated by means of determining the fractional tooth height, the more the tooth wear the less the penetration rate, equation 2.16. In order to calculate the respective tooth height, a bit record for similar bit type that has been used within the same formation is necessary.

$$f_7 = e^{a_7 X_7} = e^{a_7(-h)} \dots\dots\dots(2.16)$$

2.4.7 Hydraulic Function

The function for the hydraulic effect is defined by coefficient “a₈”. The hydraulic function represents the effects of the bit hydraulic. Jet impact force was chosen as the hydraulic parameter of interest, with a normalized value of 1.0 for f₈ at 1,000 lbf, as given in equation 2.17.

$$f_8 = e^{a_8 X_8} = \left(\frac{F_j}{1000} \right)^{a_8} \dots\dots\dots(2.17)$$

The optimization approach introduces drilling parameters selection which considers reducing the drilling costs as a function of WOB and RPM.

CHAPTER 3

METHODOLOGY

3.1 PROJECT METHODOLOGY

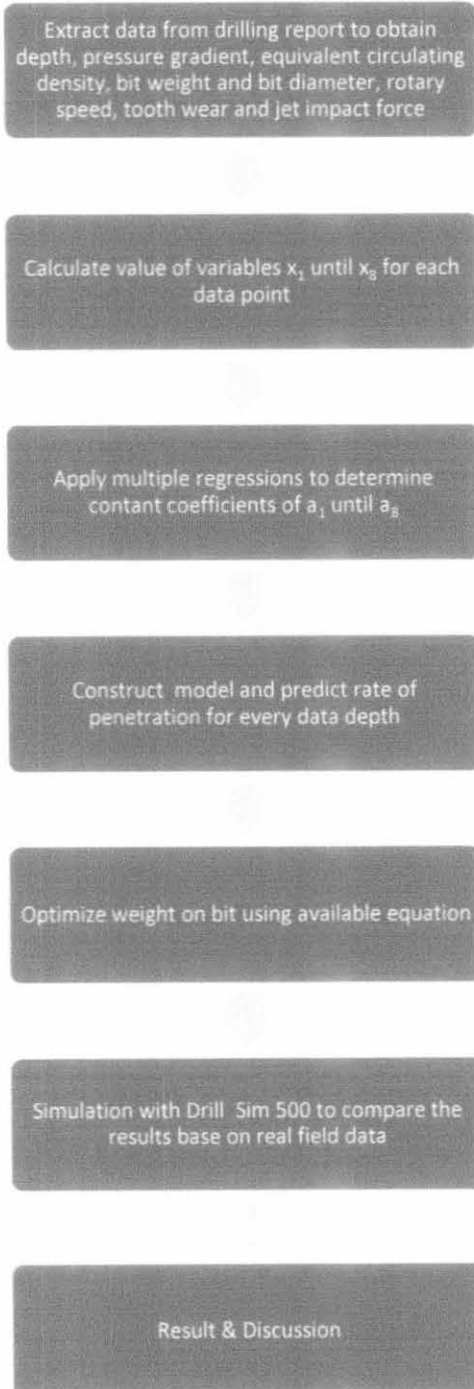


Figure 5: Project Methodology

3.2 DATA DESCRIPTION

Drilling data available for this project was acquired from a field. The subsurface geology of the offshore environment shows similarities. The lithological specification of the formation is mainly dominated by means of shale and sandstone. Table 1 gives the detailed description for the formation lithology.

Table 1: Lithology Description [21]

Depth	Description	Fluid Type
100-4000 ft	-	-
4000-6520 ft	-	-
6520-8920 ft	Sandstone, Shale	Water
8920-9560 ft	Sandstone, Shale	Gas and Oil
9560-11200 ft	Sandstone, Shale	Gas and Oil
11200-12000 ft	Sandstone, Shale	Gas and Oil
12000-13000 ft	Sandstone, Shale	Gas and Oil
13000 ft above	-	Water

The pore pressure and fracture gradient of the field is given in Table 2. Normal pressure is assumed which is 0.435 from surface to 9000ft. From 9000ft downward abnormal pressures are assume which increase in 100, 300, 600, 1000 and 1200 psi.

Table 2: Pore Pressure and Fracture Gradient [21]

Depth	Pore Pressure (psi/ft)	Fracture Gradient (psi/ft)
0-9000 ft	0.435	0.595-0.682
9000-9600 ft	0.435D + 100	0.547-0.551
9600-11200 ft	0.435 D + 300	0.547-0.551
11200-12000 ft	0.435 D + 600	0.576-0.709
12000-13000 ft	0.435 D + 1000	0.576-0.709
13000ft above	0.435D + 1200	0.576-0.709

Table 3 gives the casing and formation top details of the field used in this project. The total depth of the well is 13000ft. The conductor pipe of the wells has been installed to a depth of about 100m.

Table 3: Casing Program [21]

Depth (ftvdss)	Casing Size (in)	Grade Coupling	Weight (lb/ft)
0-325	30	ATD SQUNCH	310
325-2000	20	K55 BTC	133
2000-5900	13 3/8	N80 BTC	0-100 (72)
			100-4700 (68)
			4700-5900 (72)
5900-10800	9 5/8	P110 BTC	47
10800-13000	7	N80 VAM	29

Table 4 gives the bit and hydraulic program for the wells. The details include bit size, bit type, nozzle sizes, pump rate, mud gradient, weight on bit and rotary speed.

Table 4: Bit and Hydraulic Program [21]

Depth (tvdss)	Bit Size (Inch)	Bit Type (IADC Code)	Nozzle Size (1/32 inch)	Pump Rate (GPM)	Mud Gradient (Psi/1000)	Weight On Bit (Lbs x 1000)	RPM
325-2000	12 1/4	114	3x18	700-750	460	25-35	100-120
	26	111	3x28	950-1100		30-45	
2000-5900	12 1/4	114/PDC (PD4/BX7LM)	3x16	750-800	470	30-35	100-120 140 (PDC)
	17 1/2	114/135	3x18	800-900	470	35-40	100-120
5900-10800	12 1/4	114/PDC (PD4/ BX7LM PD5/TD290)	3x16	750-800	500	35-40 15-25	100-120 140-120
10800-13000	8 1/2	114	3x14	400-450	550	15-20	100-120
		PDC (PD4/ TD290/PD5 BX7LM)		400-500	550	15-20	120-140

Table 5 provides the design mud program which includes type of mud, mud weight, plastic viscosity and yield point.

Table 5: Mud program [21]

Depth (ftvdss)	Type	Grad (pptf)	Plastic Viscosity	Yield Point
325-2000	SLS	460	10 to 12	8 to 10
2000-5900	SLS	470	12 to 15	10 to 12
5900-10800	EA-IOEM	500	20 to 25	15 to 18
10800-13000	EA-IOEM	550	30 to 35	15 to 20

3.3 DATA PROCESS - MULTIPLE REGRESSION ANALYSIS

Figure 7 gives the multiple regressions process cycle. The first step in the process is to have the x_1 until x_8 variables calculated for each data point. The next step is to accordingly collate the calculated the variables in order to create the matrix. In the scope of this study a matrix of 8×8 is being created. Once the matrix has been calculated the same can be solved and the constant a_1 until a_8 that represent the field could be determined.

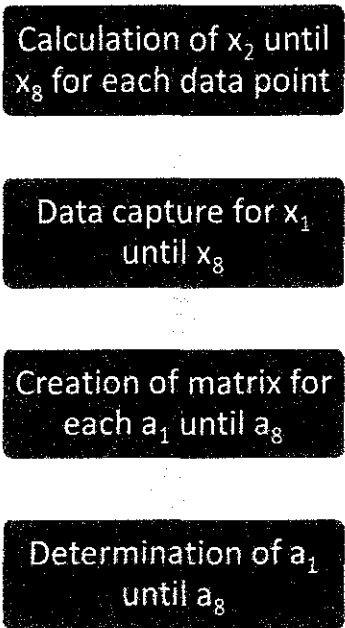


Figure 6: Multiple Regression Process Cycle

The calculation of x_2 until x_8 is using the equation of 3.1 until 3.7. This calculation is tabulated in the excel spreadsheet to make it easier for multiple regression analysis later.

$$x_2 = 10000 - D \dots \dots \dots (3.1)$$

$$x_3 = D^{0.69}(G_p - 9.0) \dots \dots \dots (3.2)$$

$$x_4 = D(G_p - P_c) \dots \dots \dots (3.3)$$

$$x_5 = \ln\left[\frac{W}{4.0}\right] \dots \dots \dots (3.4)$$

$$x_6 = \ln\left(\frac{N}{100}\right) \dots \dots \dots (3.5)$$

$$x_7 = -h \dots \dots \dots (3.6)$$

$$x_8 = \frac{Fj}{1000} \dots \dots \dots (3.7)$$

Once x_1 until x_8 have been calculated, multiple regressions are performed. Microsoft Excel has been used to process the data that available. Appendix A gives the example of written code for the multiple regression process. This way the program has been utilized to solve the constant a_1 until a_8 accurately.

3.4 RATE OF PENETRATION MODEL

The results of constant a_1 until a_8 will be use to construct the rate of penetration model based on equation 2.9. The result will be compared with the actual field to conclude the accuracy of the model.

3.5 WEIGHT ON BIT OPTIMIZATION

After the model has been constructed, several points will be selected for optimization. Optimized weight on bit that has been calculated is presented in the next chapter. The equation for optimize weight on bit for each diameter is given by equation 3.8

$$\left(\frac{W}{d_b}\right)_{Opt} = \frac{a_5 H_1 \left(\frac{W}{d_b}\right)_{max} + a_6 \left(\frac{W}{d_b}\right)_t}{a_5 H_1 + a_6} \dots\dots\dots(3.8)$$

a_5 is the weight on bit coefficient while a_6 is rotary speed coefficient which is determine during multiple regression analysis. H_1 and $(W/D_b)_{max}$ can be obtain from the recommended value of bit constants shown below.

Table 6: Recommended value of bit constant [14]

Bit Class	H_1	$(W/D_b)_{max}$
1-1 & 1-2	1.9	7
1-3 & 1-4	1.84	8
2-1 & 2-2	1.8	8.5
2-3	1.76	9
3-1	1.7	10
3-2	1.65	10
3-3	1.6	10
4-1	1.5	10
Insert	1.5	See below

Table 7: Maximum Design Weight on Bit, 1,000 lb/in [14]

Bit Class- Subclass									Insert Bits					
Bit Size	1-1	1-2	1-3	1-4	2-1	2-2	2-3	3	4	5	6	7	8	9
6 1/8		5.6	6	6.6	6.9			7.9						
6 3/4		5.7	6.1	6.6	7.1	7.2	8.5			3.1	4.4	4.5	5.2	4
7 7/8	6	6.2	6.6	7	7.5	7.6	8.7	9.4		3.5	4.5	5	5.7	4.5
8 3/4	6.2	6.5	6.8	7.2	7.8	8	9.5	10		3.7	5.1	5.2	5.8	4.7
9 7/8	6.5	6.7	7.1	7	7.6	7.7	8.9			3.6	5.1	5.1	5.9	4.6
10 5/8		6.4		7				8.8		3.5	5	5	5.8	4.5
12 1/4	5.9	6.1	6.4	6.7	7.3	7.4	8.5			3.5	4.9	4.9	5.6	4.4
14 3/4														4
15		5.3		5.8			6.3	7.4		3.4	4.7	4.8	5.4	4.3
17 1/2		5		5.7				7		3	4.2	4.2	4.8	3.8

3.6 SIMULATION

In this project, to prove the calculation of optimized value for weight on bit, Drill Simulator is used to show the result of rate of penetration based on different ranges of weight on bit. Using Drill Sim 500, all the parameters were set to match the actual field conditions. Data have been extracted to fill the required settings in the drill simulator. Several references such as Drilling Standard Handbook [20] are used to determine the standard drilling assumption for several unknowns. Figure 8 show the input settings

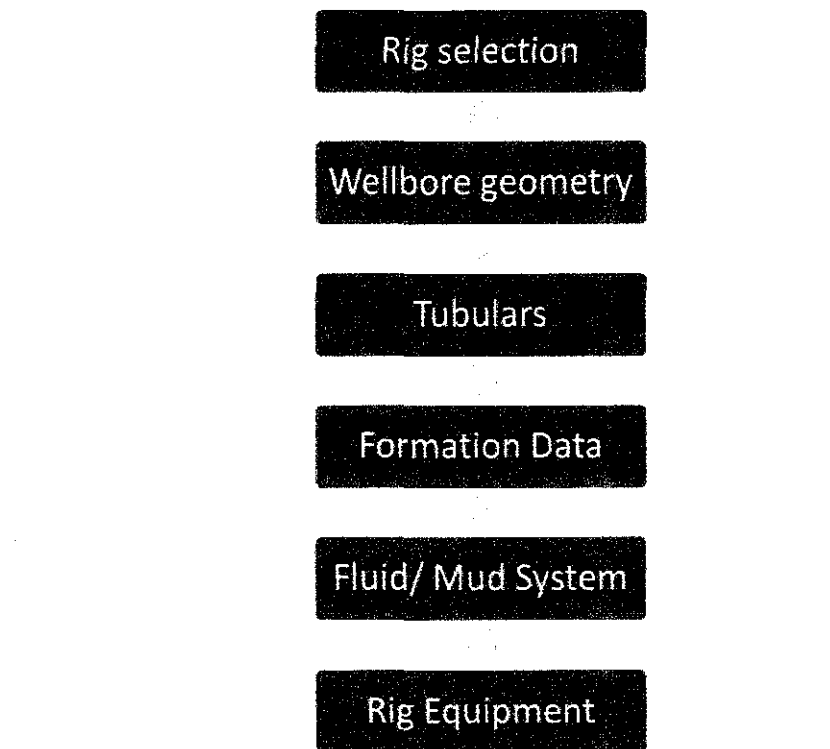


Figure 7: Drill Simulator Parameters

The simulation is started based on the actual operation of the field as the base case. When the output of simulator matching the field data, the simulation is assume to match the field condition. The simulation conditions are based on several ranges of weight on bit which is from 10000lb to 50000lb. The outputs, which are the rate of penetration, are taken as the results.

CHAPTER 4

RESULT& DISCUSSION

Table 8 shows the important parameters which include depth, drilling rate, bit weight, rotary speed, tooth wear, jet impact force, pore pressure gradient and equivalent circulating density that had been extracted from drilling report.

4.1 FIELD DATA

Table 8: Field data

Data Entry	Depth (ft)	Bit Number	Drilling Rate (ft/hr)	Bit Weight (1,000 lb/in.)	Rotary Speed (rpm)	Tooth Wear	Jet Impact Force	ECD (lb/gal)	Pore Gradient (lb/gal)
1	2150	2	171	0.82	120	-0.5	0.882	8.93	8.365
2	2155	7	20	0.57	110	-0.125	0.819	9.06	8.365
3	3591	8	160	0.82	120	-0.5	1.29	9.11	8.365
4	5190	10	82	1.63	120	-0.75	1.29	9.11	8.365
5	5872	11	49	2.45	120	-0.875	1.29	9.11	8.365
6	6000	12	43	2.45	120	-0.25	1.29	9.11	8.365
7	6080	16	64	1.63	120	-0.625	1.062	9.49	8.365
8	6322	17	36	2.45	120	-0.875	0.772	9.67	8.365
9	6592	18	27	2.85	120	-1	0.772	9.67	8.365
10	6679	19	14	0.41	120	-0.625	1.338	9.69	8.365
11	7341	20	83	1.63	180	-0.375	1.145	9.69	8.365
12	8921	21	46	1.63	180	0	1.216	9.68	8.365
13	9363	22	47	1.63	180	0	0.868	9.88	8.571
14	9652	23	19	2.85	100	-1	1.192	9.96	8.96
15	9660	24	3	2.45	65	-0.125	1.192	9.96	8.96
16	10662	25	34	1.22	180	0	1.097	9.96	8.91
17	10735	26	16	2.86	65	-0.125	1.192	9.96	8.9
18	10900	27	35	0.82	150	0	1.034	9.96	8.89
19	11214	28	12	3.27	70	-0.25	1.114	9.96	8.88
20	11224	31	5	2.94	100	-0.375	0.903	11.1	9.39
21	11481	32	26	1.76	170	0	0.975	11.02	9.37
22	12885	33	28	1.76	160	0	0.975	11.02	9.86
23	13180	34	11	1.76	130	0	0.825	10.96	10.12
24	13810	35	21	1.76	150	0	0.632	10.97	10.04
25	14300	37	15	1.76	160	0	0.632	10.95	9.98

4.2 CALCULATION OF VARIABLE x_1 UNTIL x_8

A spreadsheet was created to determine the value of x_1 until x_8 which represent variables of under compaction, normal compaction, pressure differential, weight on bit, rotary speed, bit tooth wear and jet impact force. These are based on equations 3.1 until 3.7. These values are needed before applying multiple regressions analysis to get the constants coefficients of a_1 until a_8 for the field. Table 9 shows the calculated value for the field.

Table 9: Calculation for $x_1 - x_8$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Y
1	7850	-127	-1215	-1.585	0.182	-0.500	-0.126	5.142
2	7845	-127	-1498	-1.948	0.095	-0.125	-0.200	2.996
3	6409	-180	-2675	-1.585	0.182	-0.500	0.255	5.075
4	4810	-232	-3867	-0.898	0.182	-0.750	0.255	4.407
5	4128	-253	-4375	-0.490	0.182	-0.875	0.255	3.892
6	4000	-257	-4470	-0.490	0.182	-0.250	0.255	3.761
7	3920	-259	-6840	-0.898	0.182	-0.625	0.060	4.159
8	3678	-266	-8250	-0.490	0.182	-0.875	-0.259	3.584
9	3408	-274	-8603	-0.339	0.182	-1.000	-0.259	3.296
10	3321	-277	-8850	-2.278	0.182	-0.625	0.291	2.639
11	2659	-295	-9727	-0.898	0.588	-0.375	0.135	4.419
12	1079	-338	-11731	-0.898	0.588	0.000	0.196	3.829
13	637	-236	-12256	-0.898	0.588	0.000	-0.142	3.850
14	348	-22	-9652	-0.339	0.000	-1.000	0.176	2.944
15	340	-22	-9660	-0.490	-0.431	-0.125	0.176	1.099
16	-662	-54	-11195	-1.187	0.588	0.000	0.093	3.526
17	-735	-60	-11379	-0.335	-0.431	-0.125	0.176	2.773
18	-900	-67	-11663	-1.585	0.405	0.000	0.033	3.555
19	-1214	-75	-12111	-0.202	-0.357	-0.250	0.108	2.485
20	-1224	243	-19193	-0.308	0.000	-0.375	-0.102	1.609
21	-1481	234	-18944	-0.821	0.531	0.000	-0.025	3.258
22	-2885	589	-14947	-0.821	0.470	0.000	-0.025	3.332
23	-3180	780	-11071	-0.821	0.262	0.000	-0.192	2.398
24	-3810	748	-12843	-0.821	0.405	0.000	-0.459	3.045
25	-4300	722	-13871	-0.821	0.470	0.000	-0.459	2.708

4.3 MULTIPLE REGRESSION ANALYSIS

After obtained the values of x_1 until x_8 , multiple regressions analysis was performed to obtain constant coefficients of a_1 until a_8 for the field. Table 10 shows the results of the analysis.

Table 10: Multiple regression analysis: Determination of constants a_1 - a_8 for field

Variable	Constant	Value
Drillability	a_1	3.91
Normal Compaction	a_2	9.45E-05
Under Compaction	a_3	6.86E-05
Pressure Differential	a_4	8.64E-05
Weight On Bit	a_5	0.37
Rotary Speed	a_6	2.23
Tooth Wear	a_7	0.025
Jet Impact Force	a_8	0.67

Based on the values of constant coefficients, the model for the field based on Bourgoyne and Young ROP model can be constructed as below:

$$f(x) = \text{Exp}(3.91 + 9.45(8000 - D) + 6.86 \times 10^{-5}(D^{0.69}(g_p - 9) + 8.64 \times 10^{-5} - 5D(g_p - P_c) + 0.37 \ln\left(\frac{w}{d_p^{0.02}}\right) + 2.23 \ln\left(\frac{N}{100}\right) + 0.025(-h) + 0.67\left(\frac{F_I}{1000}\right) \dots (4.1)$$

4.4 RATE OF PENETRATION

Rate of penetration for every data depth were calculated using the model that had been constructed in Equation 4.1. Table 11 shows the rate of penetration that was obtained.

Table 11: ROP determination

a_1	a_2x_2	a_3x_3	a_4x_4	a_5x_5	a_6x_6	a_7x_7	a_8x_8	Sum	Rate of penetration (ft/hr) = exp (Sum)
3.609	0.742	-0.009	-0.103	-0.591	0.407	-0.012	-0.084	3.958	52.4
3.609	0.742	-0.009	-0.129	-0.727	0.213	-0.003	-0.134	3.561	35.2
3.609	0.606	-0.012	-0.234	-0.591	0.407	-0.012	0.171	3.942	51.5
3.609	0.455	-0.016	-0.339	-0.335	0.407	-0.019	0.171	3.933	51.1
3.609	0.390	-0.017	-0.383	-0.183	0.407	-0.022	0.171	3.972	53.1
3.609	0.378	-0.018	-0.391	-0.183	0.407	-0.006	0.171	3.967	52.8
3.609	0.371	-0.018	-0.596	-0.335	0.407	-0.015	0.040	3.462	31.9
3.609	0.348	-0.018	-0.718	-0.183	0.407	-0.022	-0.173	3.249	25.8
3.609	0.322	-0.019	-0.749	-0.126	0.407	-0.025	-0.173	3.246	25.7
3.609	0.314	-0.019	-0.765	-0.850	0.407	-0.015	0.195	2.876	17.7
3.609	0.251	-0.020	-0.841	-0.335	1.312	-0.009	0.091	4.058	57.9
3.609	0.102	-0.023	-1.022	-0.335	1.312	0.000	0.131	3.774	43.6
3.609	0.060	-0.016	-1.059	-0.335	1.312	0.000	-0.095	3.476	32.3
3.609	0.033	-0.017	-1.164	-0.126	0.000	-0.025	0.118	2.428	11.3
3.609	0.032	-0.017	-1.165	-0.183	-0.961	-0.003	0.118	1.429	4.2
3.609	-0.063	-0.004	-0.980	-0.443	1.312	0.000	0.062	3.493	32.9
3.609	-0.069	-0.004	-0.981	-0.125	-0.961	-0.003	0.118	1.583	4.9
3.609	-0.085	-0.004	-1.013	-0.591	0.905	0.000	0.022	2.843	17.2
3.609	-0.115	-0.005	-1.047	-0.075	-0.796	-0.006	0.072	1.638	5.1
3.609	-0.116	-0.005	-1.833	-0.115	0.000	-0.009	-0.068	1.462	4.3
3.609	-0.140	-0.006	-1.867	-0.306	1.184	0.000	-0.017	2.457	11.7
3.609	-0.273	0.012	-1.669	-0.306	1.049	0.000	-0.017	2.405	11.1
3.609	-0.301	0.039	-1.043	-0.306	0.586	0.000	-0.129	2.455	11.6
3.609	-0.360	0.051	-0.852	-0.306	0.905	0.000	-0.308	2.739	15.5
3.609	-0.406	0.049	-0.940	-0.306	1.049	0.000	-0.308	2.747	15.6

4.5 ACTUAL AND PREDICTED ROP

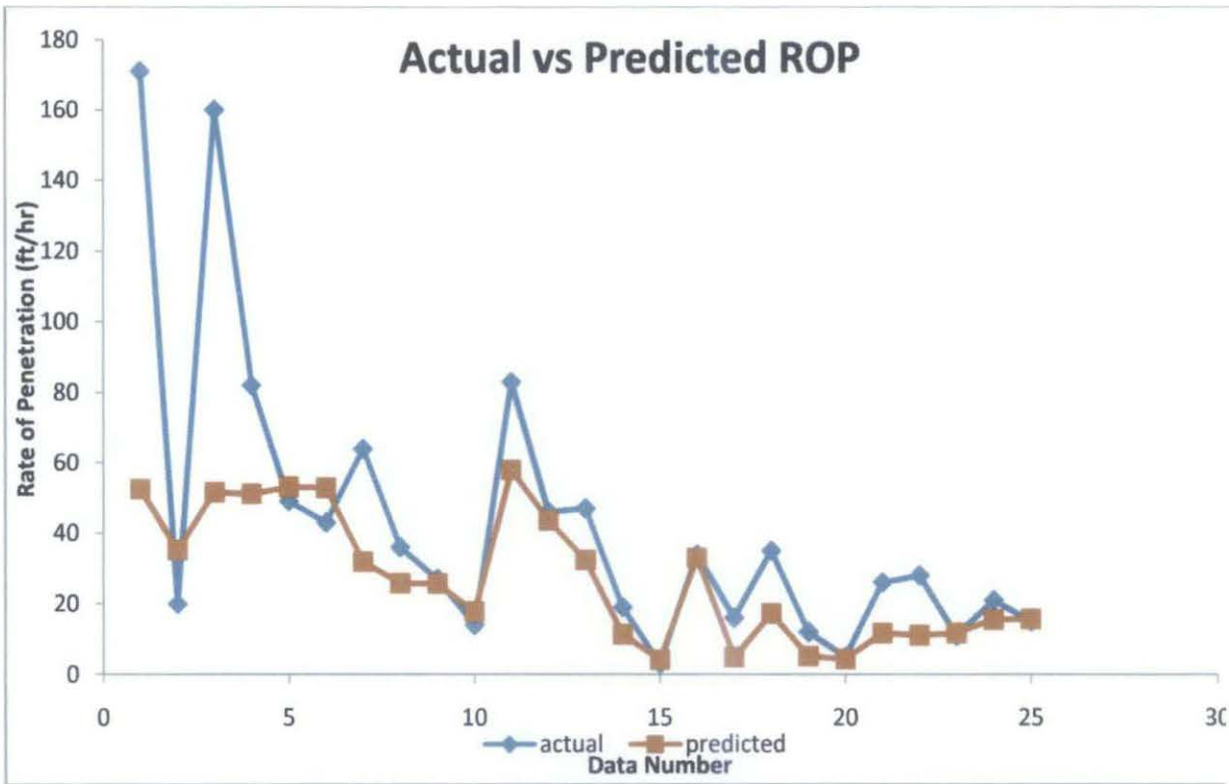


Figure 8: Actual vs. Predicted ROP

Figure 9 shows the graph of actual rate of penetration that had been taken from the actual field compared to the predicted rate of penetration using the model that had been constructed using the Bourgoyne and Young rate of penetration model. The results show accurate prediction of rate of penetration values of several data. These are shown at data number 9, 10, 12, 15, 16, 20, 23 and 25. However the prediction of rate of penetration from data number 1 until 8 doesn't show accurate value. The reason is because the statistical analysis that had been use to determine the constants doesn't have enough data that represent most of the formations. However with increased data points in the database the regressions constant gradually become better and the rate of penetration had been more accurate. Otherwise, most of the data from number 10 until 25 show comparable shape which mean that the model that had been constructed is reliable.

4.6 OPTIMIZATION

Data number 9, 10 and 15 were selected for weight on bit optimization. The selection of based on the accuracy of predicted rate of penetration and the availability of the data. They represented data at depth of 6592 ft, 6679 ft and 9660 ft. The results of weight on bit optimization were shown below. These calculations were based on the equation 3.8.

Table 12: Result of weight on bit optimization

Data Number	Depth (ft)	Rotary Speed (rpm)	Actual WOB(lb)	Actual Penetration Rate (ft/hr)	Optimized WOB(lb)
9	6592-6679	120	30000	27	23888
10	6679-7341	120	5000	14	23888
15	9660-10662	65	30000	3	8575

The table shows the optimization results for three data depth. At 6592ft, the optimize weight on bit that had been calculated is 23888lb compared to actual weight on bit of 30000lb. At 6679ft, the optimize weight on bit is also 23888lb while the actual field value is 5000lb. For data at depth 9660ft, the optimize weight on bit is 8575lb instead of 30000lb that obtained from the actual field value.

4.7 SIMULATION

To simulate the results that had been calculated from Table 12, simulations using Drill Sim 500 were conducted. Below are the results that had been obtained for data at depth 6592ft, 6679ft and 9660ft.

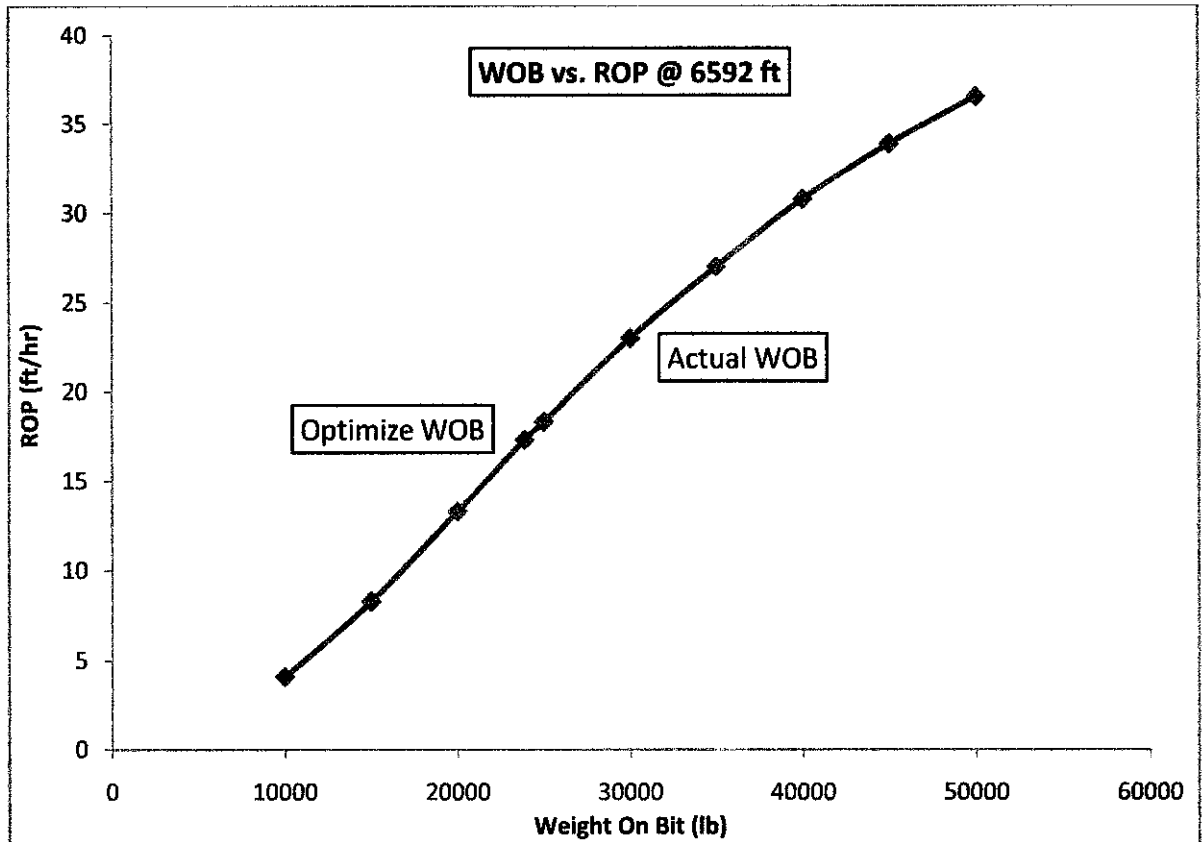


Figure 9: WOB vs. ROP for depth 6592-6679

For depth at 6592 ft, the actual weight on bit is 23888lb while the optimize weight on bit is 30000lb. From the graph, increase weight on bit has increased the values of rate of penetration. As weight on bit is increase from 15000lb to 23888lb the slope is linear, however as more weight is added, the value of slope has become curvier. This can made conclusion that the intersection between the linear line and the curvier line is the optimum value of weight on bit. Based on the calculation it is at 23888lb.

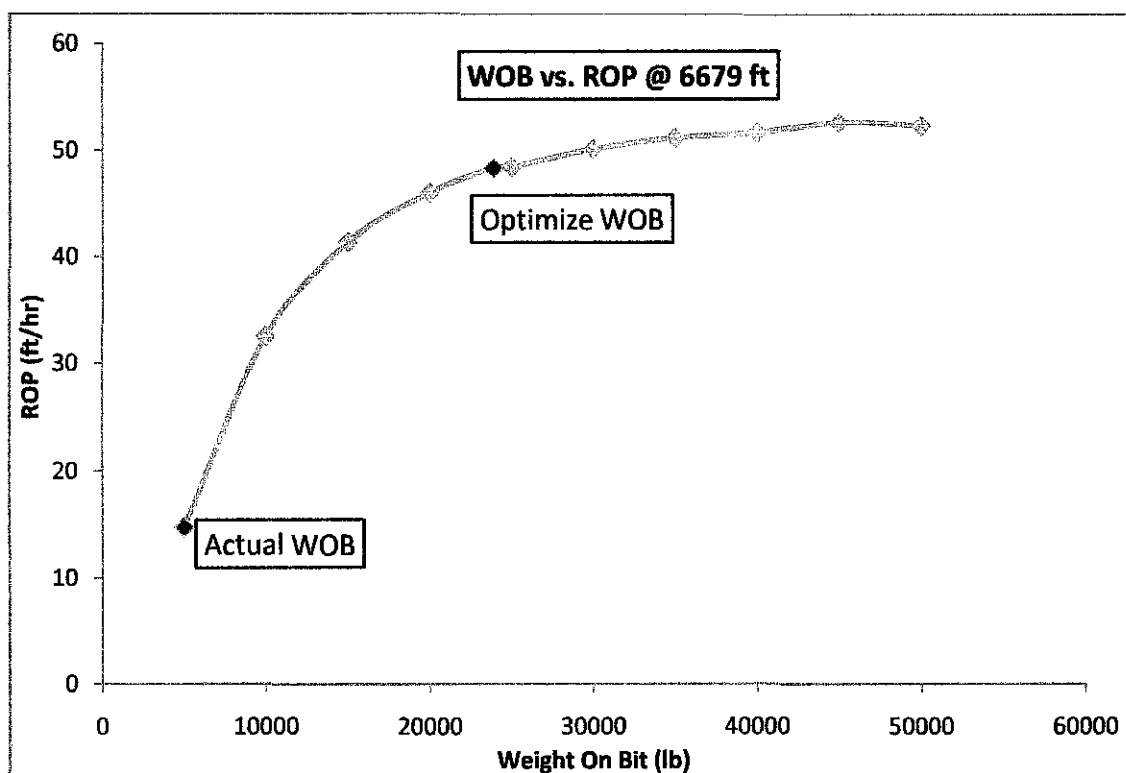


Figure 10: WOB vs. ROP for depth 6679-7341

For depth at 6679ft, the actual weight on bit are 5000lb while the optimize weight on bit are 30000lb. From the graph, increase weight on bit has increased the values of rate of penetration rapidly. As weight on bit is increase from 5000lb to 23888 lb the weight on bit has increased rapidly from 14.7 ft/hr to 48.33 ft/hr. However as more weight is added the value of rate of penetration increased in small range around 50ft/hr. This mean that during this condition, rate of cutting build up is higher than the displacement of the cutting to the surface which result in no significant increase in rate of penetration. With that reason the optimize value of 23888lb is valid as the rate of penetration is still high.

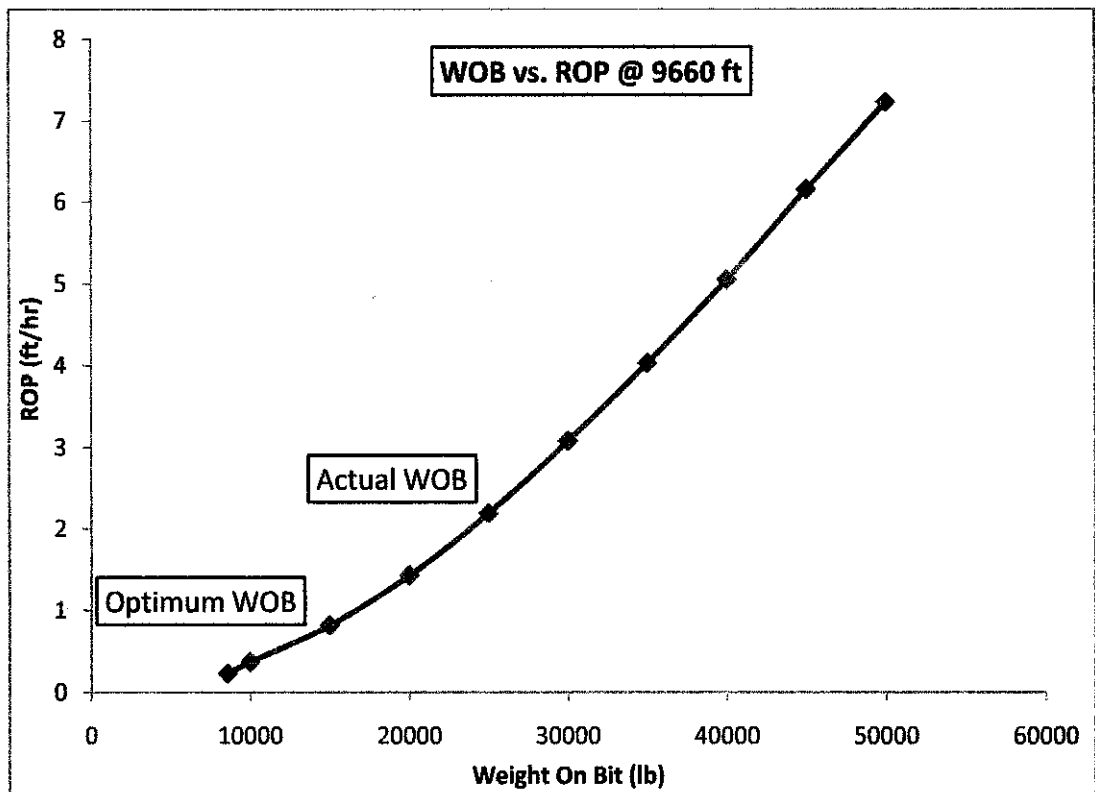


Figure 11: WOB vs. ROP for depth 9660-10662

For depth at 6592 ft, the optimum value of weight on bit is 8575lb while the actual value of weight on bit is 30000lb. From the graph, increase weight on bit has increased the values of rate of penetration. However, the ranges of rate of penetration are small from 0.23 to 7.23 ft/hr. This indicates that the drilling had encountered hard formation. For this formation, higher weight on bit could cause damage to the bit. So the optimize value of weight on bit which is 8575lb is valid as low weight on bit will reduce the bit tooth wear.

CHAPTER 5

CONCLUSION

The results of optimization show significance results for this project. Determination of optimum weight on bit is very important in drilling operation as this parameter can be change during drilling operation. The optimization of weight on bit will optimize the whole drilling operation as a whole. Increasing rate of penetration will reduce the time need for drilling those reduces the cost for drilling operations.

1. The constants a_1 until a_8 which represent formation strength, under compaction, normal compaction, pressure differential, weight on bit, rotary speed, bit tooth wear and jet impact force has been achieved using multiple regressions analysis.
2. Bourgoyne and Young model produce reliable rate of penetration model. Data number 9,10,12,15,16,20,23 and 25 were predicted accurately compare with the actual rate of penetration obtained from the field.
3. Optimization for weight on bit found that for depth at 6592ft optimize weight on bit is 23888lb compare to 30000lb, at 6679 ft optimize value of weight on bit is 23888lb compare to 5000lb. For 9660ft optimize value is 8575lb compare to 30000lb.

The results of this project provide guidance for next drilling operation near the drilled well. The optimize values can be used as reference to obtain optimum drilling performance those reduce drilling cost.

CHAPTER 6

RECOMMENDATIONS

Optimization of drilling operation has provide significant reduction not only to reduce the time for drilling but to have an optimize operation that could save drilling cost. This is important as nowadays drilling cost is very high due to complex drilling operation.

1. As drilling operation is very crucial and costly it is important to have more researches in this field to make sure more improvements can be achieve. Research should be done extensively to make sure the develop model could replace the old model to suit the current drilling condition.
2. In future work, several parameters that were not include in the model but relate to rate of penetration should be include such as drilling fluid details, solids content and efficiency of the rig equipment/material. These will make sure the model is reliable thus the optimization is more accurate.
3. The used of multiple regression analysis for prediction of coefficients could be replaced by more sophisticated and modern statistical methods. Recent studies include several statistical methods that can be applied to obtain more accurate coefficients such as Genetic Algorithm (G.A) or Artificial Neural Network.

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APPENDICE

Appendix A: Example of Multiple Regression Analysis Code

```

Model = "= "

Model = Model & ""Y""
Model = Model & " & "
Model = Model & "" = ""

'The information for the model statement is taken from the _
worksheet and not hard coded.
For i = 1 To p
    If Intercept = i Then
        temp = " & " & "round(B19,2)"
    Else
        temp = " & if(sign(B" & 18 + i & ")=-1, "" "", "" + "" )" & _
            " & " & "Round(B" & 18 + i & ", 2)" & " & " & _
            "" "" & " & " & "A" & 18 + i
    End If
    Model = Model & temp
Next i

Call Progress(0.75) 'update procedure's progress

*****
*****          OUTPUT          *****
*****

'Output in new worksheet
'Check workbook for a worksheet named "Regression"
For Each wks In Application.Worksheets
    If wks.name = "Regression" Then wks.Delete
Next

'Place new worksheet after the last worksheet in the workbook
cntsheets = Application.Sheets.Count
Set newsheet = Application.Worksheets.add(after:=Worksheets(cntsheets))
newsheet.name = "Regression"
FinalCol = 0

Call Progress(0.8) 'update procedure's progress

'Get the sheet name-either new or existing
ShtName = Application.ActiveSheet.name

With Application
    'Place the data in the worksheet along with variable names
    .Cells(1, 13 + p).Value = Varnames(1)
    .Range(Cells(2, 13 + p), Cells(n + 1, 13 + p)).Value = y
    For i = 1 To p
        If Intercept = i Then
            .Cells(1, 13 + p + i).Value = "Intercept"
        Else
            .Cells(1, 13 + p + i).Value = Varnames(i + 1 - Intercept)
        End If
    End If
End With

```

```

Next i
.Range(Cells(2, 14 + p), Cells(n + 1, 13 + p + p)) = Data

'Insert the range formula for the X'Xinv
.Cells(1, 8).Value = "X'X inverse"
.Range(Cells(2, 8), Cells(1 + p, 7 + p)).FormulaArray = _
    "=MINVERSE(MMULT(TRANSPOSE(RC[" & 6 + p & _
    "]:R[" & n - 1 & "]C[" & 5 + p + p & "]),RC[" & 6 + p & _
    "]:R[" & n - 1 & "]C[" & 5 + p + p & "]))"

'Insert the fomulae for the variance-covariance matrix
.Cells(2 + p, 8).Value = "Variance-covariance matrix"
.Range(Cells(3 + p, 8), Cells(p * 2 + 2, 7 + p)).FormulaR1C1 = _
    "=R8C4*R[-" & 1 + p & "]C"

'Build the correlation matrix using the 'Correl' function
'Must apply the function to all combinations to get lower _
triangular of correlation matrix--get other half by symmetry
.Cells(3 + 2 * p, 8).Value = "Correlation matrix"
For i = 1 To p - Intercept
    .Cells(3 + 2 * p + i, 7 + i).Value = 1#
    For j = i + 1 To p - Intercept
        .Cells(3 + 2 * p + j, 7 + i).FormulaR1C1 = _
            "=Correl(R2C" & 13 + p + Intercept + i & _
            ":R" & n + 1 & "C" & 13 + p + Intercept + i & ", " & _
            "R2C" & 13 + p + Intercept + j & ":R" & n + 1 & "C" & _
            13 + p + Intercept + j & ")"
        .Cells(3 + 2 * p + i, 7 + j).FormulaR1C1 = _
            "=R[" & j - i & "]C[-" & j - i & "]"
    Next j
Next i

'Calculate the inverse of the correlation matrix
Cells(4 + 2 * p + p - Intercept, 8).Value = "Inverse Correlation Matrix"
.Range(Cells(5 + 2 * p + p - Intercept, 8), _
    Cells(4 + 2 * p + 2 * (p - Intercept), 7 + p -
Intercept)).FormulaArray = _
    "=MINVERSE(R" & 4 + 2 * p & "C8:R" & 3 + 2 * p + p -
Intercept & _
    "C" & 7 + p - Intercept & ")"

Call Progress(0.85) 'update procedure's progress

'Output ANOVA table
.Cells(1, 1).Value = "Regression Analysis of " & Varnames(1)
.Cells(1, 1).Font.Bold = True
.Cells(3, 1).Value = "Regression equation:"
On Error Resume Next
.Cells(3, 2).Value = Model
On Error GoTo 0
.Cells(5, 2).Value = "Sum of"
.Cells(5, 3).Value = "Degrees of"
.Cells(5, 4).Value = "Mean"
.Cells(6, 1).Value = "Source of Variation"
.Cells(6, 2).Value = "Squares"
.Cells(6, 3).Value = "Freedom"
.Cells(6, 4).Value = "Square"
.Cells(6, 5).Value = "F"
.Cells(6, 6).Value = "P-value"

```

```

'Output fitted values
.Cells(1, 9 + p).Value = "Fits"
.Range(Cells(2, 9 + p), Cells(n + 1, 9 + p)).FormulaR1C1 = _
    "=MMULT(RC[5]:RC[" & 4 + p & "],R19C2:R" & 18 + p & "C2)"

'Output residuals
.Cells(1, 10 + p).Value = "Resids"
.Range(Cells(2, 10 + p), Cells(n + 1, 10 + p)).FormulaR1C1 = _
    "=RC[3]-RC[-1]"

'Output regression sum of squares
.Cells(7, 1).Value = "Regression"
.Cells(7, 2).FormulaR1C1 = "=R[2]C-R[1]C"
.Cells(7, 3).Value = p ~ Intercept
.Cells(7, 4).FormulaR1C1 = "=RC[-2]/RC[-1]"
.Cells(7, 5).FormulaR1C1 = "=RC[-1]/R[1]C[-1]"

'Output error sum of squares
.Cells(8, 1).Value = "Error"
.Cells(8, 2).FormulaR1C1 = _
    "=SUMSQ(R2C" & 10 + p & ":R" & n + 1 & "C" & 10 + p & ")"
.Cells(8, 3).Value = n - p
.Cells(8, 4).FormulaR1C1 = "=RC[-2]/RC[-1]"

'Output total sum of squares
.Cells(9, 1).Value = "Total"
If Intercept = 1 Then
    .Cells(9, 2).FormulaR1C1 = _
        "=DEVSQ(R2C" & 13 + p & ":R" & n + 1 & "C" & 13 + p & ")"
Else
    .Cells(9, 2).FormulaR1C1 = _
        "=SUMSQ(R2C" & 13 + p & ":R" & n + 1 & "C" & 13 + p & ")"
End If

'Output error degrees of freedom
.Cells(9, 3).Value = n - Intercept

'Output RMSE
.Cells(11, 2).Value = "s"
.Cells(11, 3).FormulaR1C1 = "=SQRT(R[-3]C[1])"
.Cells(11, 3).NumberFormat = "0.0000"

'Output Rsq only with intercept model
If Intercept = 1 Then
    .Cells(12, 2).Value = "R-sq"
    .Cells(12, 3).FormulaR1C1 = "=R[-5]C[-1]/R[-3]C[-1]"
    .Cells(12, 3).NumberFormat = "0.00%"
    .Cells(13, 2).Value = "R-Sq(adj)"
    .Cells(13, 3).FormulaR1C1 = "=1-R8C4/(R9C2/R8C3)"
    .Cells(13, 3).NumberFormat = "0.00%"
End If

'Output table of coefficient estimates, etc.
.Cells(16, 1).Value = "Parameter Estimates"
.Cells(18, 1).Value = "Predictor"
.Cells(18, 2).Value = "CoefEst"
.Cells(18, 3).Value = "Std Error"
.Cells(18, 4).Value = "t value"

```

```

.Cells(18, 5).Value = "P-value"

'General formatting
'Draw lines on ANOVA table
Range(.Cells(4, 1), Cells(4, 6)) _
    .Borders(xlEdgeBottom).LineStyle = xlContinuous
Range(.Cells(6, 1), Cells(6, 6)) _
    .Borders(xlEdgeBottom).LineStyle = xlContinuous
Range(.Cells(9, 1), Cells(9, 6)) _
    .Borders(xlEdgeBottom).LineStyle = xlContinuous
'Draw line for table of coefs, se, VIFs, t & p statistics
Range(.Cells(18, 1), Cells(18, 5)) _
    .Borders(xlEdgeBottom).LineStyle = xlContinuous

.Columns(1).ColumnWidth = 18
.Columns(3).ColumnWidth = 11
.Columns(4).ColumnWidth = 9.75
.Range(Cells(7, 5), Cells(7, 6)).NumberFormat = "0.0000"

'Output the coefficient estimates
.Range(Cells(19, 2), Cells(18 + p, 2)).FormulaArray = _
    "=MMULT(R2C8:R" & 1 + p & "C" & 7 + p & _
    ",MMULT(TRANSPOSE(R2C" & 14 + p & ":R" & _
    n + 1 & "C" & 13 + 2 * p & ")," & _
    "R2C" & 13 + p & ":R" & n + 1 & "C" & 13 + p & "))"

'Ouput SEs, t values, pvalues, and VIFs
For i = 1 To p
    'Output variable names
    If i = 1 Then
        If Intercept = 1 Then
            .Cells(i + 18, 1).Value = "Constant"
        Else
            .Cells(i + 18, 1).Value = VarNames(i + 1)
        End If
    Else
        .Cells(i + 18, 1).Value = VarNames(i)
    End If
    .Cells(i + 18, 2).NumberFormat = "0.0000"

    'Output standard errors
    .Cells(i + 18, 3).FormulaR1C1 = _
        "=SQRT(R" & 2 + p + i & "C[" & 4 + i & "])"
    .Cells(i + 18, 3).NumberFormat = "0.0000"
    .Cells(i + 18, 4).NumberFormat = "0.0000"
    .Cells(i + 18, 5).NumberFormat = "0.0000"

    'Output VIFs
    If i > 1 And Intercept = 1 And p > 2 Then
        .Cells(18, 6) = "VIFs"
        .Cells(i + 18, 6).FormulaR1C1 = _
            "=R" & 3 + 3 * p ~ Intercept + i & "C[" & i & "]"
        .Cells(i + 18, 6).NumberFormat = "0.0000"
        .Cells(18, 6).Borders(xlEdgeBottom) _
            .LineStyle = xlContinuous
    End If
Next i

```

```

Call Progress(0.9)      'update procedure's progress

'Write note detailing the use of observations
If IsEmpty(Missing) Then
    .Cells(i + 19, 1) = n & _
        " observations were used in the analysis."
Else
    .Cells(i + 19, 1) = n & _
        " observations were used in the analysis."
'Two statements to get the verb tense correct
If UBound(Missing, 1) = 1 Then
    .Cells(i + 20, 1) = UBound(Missing, 1) & _
        " observation was excluded due to missing values."
Else
    .Cells(i + 20, 1) = UBound(Missing, 1) & _
        " observations were excluded due to missing values."
End If
End If

'*****
' Diagnostic Calculations
'*****

'Output the value of the determinant of the correlation matrix
.Cells(11, 4) = "Determinant"
.Range(Cells(11, 5), Cells(11, 5)).FormulaArray = _
    "=MDETERM(R" & 4 + 2 * p & _
    "C8:R" & 3 + 2 * p + p - Intercept & _
    "C" & 7 + p - Intercept & ")"

Call Progress(0.95)      'update procedure's progress

'Durbin-Watson statistic
.Cells(1, 11 + p).Value = "Durbin-Watson"
.Range(Cells(3, 11 + p), Cells(n + 1, 11 + p)).FormulaR1C1 = _
    "=(RC[-1]-R[-1]C[-1])^2"
.Cells(12, 4) = "DW"
.Cells(12, 5).FormulaR1C1 = _
    "=SUM(R3C" & 11 + p & ":R" & n + 1 & "C" & 11 + p & ")/R8C2"
.Cells(12, 5).NumberFormat = "0.00"

'Output processing time
.Worksheets(ShtName).Cells(22 + p, 1) = _
    "Computational time: " & .Round(Timer - Time, 2) & " seconds."

Call Progress(1)      'update procedure's progress

'resume worksheet calculations
.Calculation = xlCalculationAutomatic

'Calculate probabilities after worksheet calculations _
have been set to automatic
't values
.Range(Cells(19, 4), Cells(18 + p, 4)). _
    FormulaR1C1 = "=RC[-2]/RC[-1]"
'p values
.Range(Cells(19, 5), Cells(18 + p, 5)). _
    FormulaR1C1 = "=(1-TCDF(abs(RC[-1]),R8C3))*2"

```



```

        'F value
        .Cells(7, 6).FormulaR1C1 = "=1-FCDF(RC[-1],RC[-3],R[1]C[-3])"

    End With

Unload Me
End

EndProc:
Application.Calculation = xlCalculationAutomatic      'resume worksheet
calculations
MsgBox ("Procedure has encountered a fatal error and will terminate. " & _
        "Error code: " & Err)

Unload Me
End Sub

Sub Progress(Pct)
    'This sub updates the width of the bar moving across the _
    progress indicator frame and the % complete caption
    With Me
        .Progress_Frame.Caption = FormatPercent(Pct, 0)
        .ProgressBar.Width = Pct * .Progress_Frame.Width
        .Repaint
    End With
End Sub

```